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THE PREDICTION OF WALL TEMPERATURE IN THE PRESENCE OF FILM COOLING

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Abstract—A procedure has been developed for the prediction of adiabatic-wall temperature and heat-transfer coefficient downstream of two-dimensional, film cooling slots. Comparisons between predicted and measured values demonstrate that the method satisfactorily predicts the influence of velocity ratio, density gradient, longitudinal pressure gradient, slot-lip boundary-layer thickness, and the ratio of lip thickness to slot height. The procedure uses a modified form of the Spalding–Patankar scheme to solve the two-dimensional, boundary-layer equations and embodies particular forms of the Prandtl mixing length and effective Prandtl number.

Although the procedure is satisfactory for two-dimensional slot configurations with controlled flow conditions, it is shown that the prediction of combuster-wall-temperature distributions is still some way off.

NOMENCLATURE

h_1, convective heat-transfer coefficient at inner surface of flame tube [W/m² degC];
h_2, convective heat-transfer coefficient at outer surface of flame tube [W/m² degC];
κ, constant in the law of the wall;
K_p, pressure gradient parameter [$\equiv \nu/u_G^2 \, du_G/dx$];
l, mixing length [m];
l_w, characteristic length of the wake [m];
L, value of x over which Λ is evaluated, equation (9) [m];
k_C, thermal conductivity of slot fluid [W/m degC];
$\dot{m}_I''$, mass flux across I boundary [kg/m² s];
$\dot{m}_E''$, mass flux across E boundary [kg/m² s];
Nu_C, Nusselt number [(hy_C/k_C)];
p, static pressure [N/m²];
$\dot{q}_{gen}''$, heat flux generated by heating element [W/m²];
R_C, slot Reynolds number $\left[\equiv \dfrac{\rho_C \bar{u}_C y_C}{\mu_C}\right]$;
t, slot lip thickness [m];
T, absolute temperature [°K];
u, mean velocity in x-direction [m/s];
u_{max}, velocity maximum [m/s];
u_w, characteristic velocity of the wake [m/s];
$\bar{u}_C$, mean velocity at slot exit $\dfrac{1}{y_C}\int_0^{y_C} u \, dy$ [m/s];
u_G, free-stream velocity [m/s];
v, mean velocity in the y-direction [m/s];
x, distance from slot exit [m/s];
y_C, slot height [m];
$y_{C,G}$, slot-lip boundary-layer thickness;
α_W, flame absorptivity at wall temperature;
Γ_0, effective diffusivity, Fig. 1 [kg/m-s];
Γ_{add}, additive diffusivity, Fig. 1 [kg/m-s];
ε_G, emissivity of the flame;
ε_0, emissivity of the combustion chamber casing;

ε_W, emissivity of flame-tube wall;
η, effectiveness based on general conserved property ϕ
$$\left[\equiv \frac{\phi_{\mathrm{ad},W} - \phi_G}{\phi_C - \phi_G}\right];$$
η_{EXP}, experimental value of effectiveness;
η_{PRD}, predicted value of effectiveness;
Λ, a measure of deviation equation (9);
$\bar{\Lambda}$, a mean value of Λ;
μ, laminar viscosity [Ns/m^2];
μ_{eff}, effective viscosity [Ns/m^2];
ν, kinematic viscosity [m^2/s];
ξ, coefficient in equation (10);
ρ, fluid density [kg/m^3];
σ_{eff}, effective Prandtl or Schmidt number;
σ_B, Stefan–Boltzman constant [W/m^2-°K^4];
τ, shear stress in fluid [N/m^2];
ϕ, a conserved property;
ψ, stream function defined by equation (2);
ω, non-dimensional stream function, equation (3).

Subscripts
ad, pertaining to adiabatic wall;
C, pertaining to slot exit;
E, external edge of boundary layer;
I, internal edge of boundary layer;
G, pertaining to the free stream;
W, pertaining to the wall.

1. INTRODUCTION

RECENTLY reported measurements in film cooling situations together with the equally recent procedure for solving the differential equations appropriate to boundary-layer flows, have made it possible to devise a more general method for predicting the properties of such flows. Thus, the present paper reports a procedure for predicting mean-flow properties downstream of a two-dimensional film-cooling slot and demonstrates, by comparison with experimental data, that the procedure allows accurate predictions over a wide range of velocity ratios, density ratios, longitudinal pressure gradients and two-dimensional slot configurations. This procedure represents another step forward in the long-term objective to provide a means of predicting important flow properties downstream of three-dimensional film-cooling slots.

Previous attempts to predict film-cooling performance have been restricted in the range of variables for which they were valid. These restrictions were caused partly by lack of knowledge of the important parameters and partly by the form of the procedures themselves. A brief review of the current state of knowledge, of the important parameters and prediction methods is included in the succeeding paragraphs, and will be referred to later in the paper.

Many experimental investigations, e.g. [1–7], of the dependence of the adiabatic-wall (or impervious-wall) effectiveness on the velocity ratio, $\bar{u}_C/u_G$, have been reported. In contrast, there have been only a few [1, 2, 8] investigations of the dependence of local heat transfer coefficients. The ratio t/y_C has been shown, under certain conditions, to greatly influence the effectiveness [6, 8, 9] but not the heat transfer coefficient [8]. Density ratio has a considerable influence on the effectiveness [7, 10] and longitudinal pressure gradients have little effect upon effectiveness unless the pressure gradient is strong enough to laminarise or separate the boundary layer [11–14]: there is no experimental evidence of the influence of density ratio and only slight evidence of the influence of pressure gradient [12] on the heat transfer coefficient. The thickness of the boundary layer on the upper surface of the slot lip has been shown to have little influence unless t/y_C is small [15–18]. The data reported in these papers [1–18] make possible the testing of a prediction procedure over a wide range of variables.

The large number of correlation equations devised for film-cooling situations can be typified by that due to Spalding [19], i.e.

$$\eta = 1, \qquad X < 7,$$

$$\eta = 7/X, \quad X > 7,$$

where

$$X = 0{\cdot}91 \left[\frac{u_G x}{\bar{u}_C y_C}\right]^{0\cdot 8} R_C^{-0\cdot 2} + 1{\cdot}41 \left\{ \left|1 - \frac{u_G}{u_C}\right| \frac{x}{y_C} \right\}^{0\cdot 5}. \tag{1}$$

This equation, in common with all correlation equations, is valid only within the range of variables upon which it is based. Thus, the above equation, and most of its predecessors, cannot be used in the presence of density gradients; for velocity ratios in the vicinity of unity; or to predict variables other than the adiabatic-wall effectiveness. Subsequent attempts [20, 21] to devise more general procedures, based on integral forms of the conservation equations, were abandoned in favour of procedures based on differential forms of the conservation equations. Two procedures have been developed simultaneously: the one presented here is based on the parabolic boundary-layer form of the differential, conservation equations; the second [18, 22] is based on the elliptic time average Navier–Stokes equations and permits the prediction of recirculation. Both procedures are embodied in computer programs.

The experiments reported in [23] showed that normal pressure gradients and recirculation can exist in the region close to the slot exit. In this region, therefore, the parabolic equations are not entirely valid and the elliptic equations should, strictly speaking, be employed. However, since this region is of the order of 15 slot heights in length and the range of interest may extend to several hundred slot heights, depending on the application, it is desirable to determine what can be achieved with the boundary-layer equations. In addition, the solution of the parabolic equations is more than 10 times faster than the solution of the elliptic equations; this economy of computational time is a further important reason for solving the parabolic equations.

The present paper is almost entirely devoted to two-dimensional flows, in contrast to most film-cooling slots used in practice. It is useful to note, however, that prediction procedures for three-dimensional slots are entirely based upon empirical correlations and are restricted to very narrow ranges of geometrical configurations. The two-dimensional nature of the present prediction procedure is a serious limitation although it is much more general than any presently available alternative. The extent of this limitation is discussed, with particular reference to the design of combustor film-cooling slots, in a later section of this paper.

The basic features of the present procedure have been described by Spalding and Patankar [24]. In a subsequent paper [7], the application of the procedure to the special case of an impervious wall, was described. There is no need to repeat details previously described and the next section does no more than present the essential features of the differential equations, the flux laws and the solution procedure. Section 3 presents the particular forms of flux laws employed here, and the evidence for these choices; this evidence is presented separately for the upstream and downstream regions and the section is divided accordingly. Section 4 attempts to display the generality and accuracy of the procedure by presenting predictions, and making comparisons with experimental data, of the adiabatic-wall effectiveness and heat-transfer coefficient. The application of the procedure to combustor film-cooling is described and discussed in section 5. The last section, 6, summarises the achievements of the present paper and suggests guidelines for future research designed to render the prediction procedure more applicable to combustion chamber design.

2. RELEVANT EQUATIONS

The equation for conservation of mean momentum of a two-dimensional, boundary-layer flow may be written in the form:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{1}{\rho}\frac{\mathrm{d}\tau}{\mathrm{d}y}. \tag{2}$$

Introducing the stream function, defined by:

$$\rho u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad \rho v = -\frac{\partial \psi}{\partial x} \tag{3}$$

and its dimensionless form:

$$\omega = \frac{\psi - \psi_E}{\psi_I - \psi_E}, \tag{4}$$

equation (1) may be rewritten as:

$$\frac{\partial u}{\partial x} + (a + b\omega)\frac{\partial u}{\partial \omega} = \frac{\partial}{\partial \omega}\left(c\frac{\partial u}{\partial \omega}\right) + d \tag{5}$$

where

$$a \equiv \dot{m}_I''/(\psi_E - \psi_I)$$
$$b \equiv (\dot{m}_E'' - \dot{m}_I'')/(\psi_E - \psi_I)$$
$$c \equiv \rho u \mu_{\text{eff}}/(\psi_E - \psi_I)^2$$
$$d \equiv -(\mathrm{d}p/\mathrm{d}x)/\rho u.$$

in this form, i.e.

$$\frac{\partial \phi}{\partial x} + (a + b\omega)\frac{\partial \phi}{\partial \omega} = \frac{\partial}{\partial \omega}\left(c\frac{\partial \phi}{\partial \omega}\right) + d \tag{6}$$

where ϕ is a general dependent variable and c and d have the general forms:

$$c \equiv \rho u \mu_{\text{eff}}/\{(\psi_E - \psi_I)^2 \, \sigma_{\text{eff}}\}$$
$$d \equiv \Phi/\rho u.$$

The solution procedure of Spalding and Patankar [24] was devised to solve equations of the form of equation (5) and is used here because of its economy and generality. The $x - \omega$ coordinate system ensures that the computations are carried out only between values of ω of zero and unity and an empirical entrainment law ensures that this is the region where flux gradients are finite.

The term, c, includes the diffusion coefficient appropriate to the conversed property, ϕ, and, for laminar flows, this is readily specified. For turbulent flow, however, an appropriate hypothesis is necessary. The present choice for the coefficient of momentum diffusion is based on Prandtl's mixing length:

$$\mu_{\text{eff}} = \rho l^2 \left|\frac{\partial u}{\partial y}\right|. \tag{7}$$

An effective Prandtl number and Schmidt number are specified for the equations of total enthalpy and species; they are here considered to be identical, implying an effective Lewis number of unity. The specification of mixing length and effective Prandtl/Schmidt number is discussed in the next section.

3. PHYSICAL HYPOTHESES AND THEIR JUSTIFICATION

The choice of the mixing-length form of effective viscosity was made because it represents the simplest form of effective viscosity which can be used for wall-jet and wall-wake flows. More complicated forms are possible but it is desirable to test the validity of the simplest model and, in any case, the available data do not permit the testing of higher-order models over the range of variables of interest in this paper. The choice of a single equation for species and total-enthalpy conservation was made for similar reasons; once again, available data do not warrant the use of a two-equation model or, indeed, of complex forms of the effective Prandtl or Schmidt number.

Two approaches have been taken to obtaining specific forms of mixing length and Prandtl/Schmidt number. The first, the direct approach, was to examine all available experimental information from which the mixing length and Prandtl/Schmidt number profiles could be deduced. The second, the indirect approach, was to use the prediction procedure with assumed forms of the mixing length and Prandtl/Schmidt number. Both approaches were taken in an earlier paper, [25], where it was concluded that the optimum forms of mixing length and Prandtl or Schmidt number were:

$$\left.\begin{aligned} &l = \kappa y = 0{\cdot}4187y, \\ &0 < y < 0{\cdot}09 y_G/\kappa \\ &l = 0{\cdot}09, \\ &0{\cdot}09 y_G/\kappa < y < y_G \end{aligned}\right\} \tag{8}$$

and $\sigma_{eff} = 1{\cdot}75 - 1{\cdot}25y/y_G$, respectively.

Further examination, e.g. [26], has supported the choice of value of κ but not the distribution of σ_{eff}. The direct information is insufficiently precise to define σ_{eff} and, hence, a specific choice must be based on indirect evidence. This indirect evidence should be considered only in the downstream region of the flow if it is to be assessed in conjunction with a parabolic prediction procedure. This is done in the next subsection. Thereafter, modifications are described which permit the mixing length and Prandtl/Schmidt number distributions to be used in the upstream region in conjunction with the parabolic procedure. This is necessarily an approximate procedure which can be justified on grounds of simplicity and, in spite of the empiricism required, will be shown to be much more successful than previous attempts to predict flows of this type.

3.1 *The downstream region*

The computer program previously used in [25] and embodying the numerical-computational procedure of [24], was used to effect the comparisons discussed in this section. The mixing-length distribution was that given in equation (8) and, in cases where the presence of zero-velocity gradient caused the effective diffusivity to become zero, the effective diffusivity was taken as the value corresponding to a straight line between the two neighbouring effective diffusivity maxima; this bridging procedure was shown in [25] to have no significant influence on values of mean velocity.

The minimum value of the quantity:

$$\Lambda^2 = \frac{1}{L}\int_0^L (\eta_{PRD} - \eta_{EXP})^2\,dx \qquad (9)$$

was used in judging the quality of predictions. This quantity was selected since it bases the selection procedure on the adiabatic-wall effectiveness which is the property of direct relevance to this paper. A mean value of Λ, i.e.

$$\overline{\Lambda}^2 \equiv \sum_i^N \Lambda_i^2 L_i / \sum_i^N L_i,$$

was obtained for each of the three σ_{eff} distributions,

$$\sigma_{eff} = 0{\cdot}5$$

$$\sigma_{eff} = 1{\cdot}0$$

and

$$\sigma_{eff} = 1{\cdot}75 - 1{\cdot}25y/y_G.$$

The test data, the resulting values of Λ, evaluated at x/y_C of 100 $\overline{\Lambda}$ and the maximum deviation occurring between x/y_C of 20 (the initial condition) and 100 are indicated on Table 1. From this table it can easily be seen that a value of effective Prandtl/Schmidt number of unity gives the best result by a large margin. The test data were selected to cover a practical range of variables in zero pressure gradient and to provide measured profiles of velocity and temperature or concentration at or around x/y_C of 20. The selection was made in order not to bias the result too greatly towards the measurements of the present authors, the largest number of results with known boundary conditions available to the authors being their own.

The conclusion that a value for effective Prandtl/Schmidt number of unity is better than a linear distribution from 1·75 at the wall to 0·5 in the free stream is at odds with the conclusion of [25]. The reason for this lies partly in the data selected for the test and partly on a modification to the Couette flow assumption made in [25]. The consequences of this assumption were (a) that conservation of species was not precisely observed and (b) it caused the non-dimensional concentration profile to bulge outwards due to the incorrect slope at the first grid interval from the wall. Subsequently, this procedure was changed to permit the correct consideration of convection in the half-interval at the wall. The details of this modification are described in [27].

Table 1. Data used to assess effective Prandtl/Schmidt number distributions

No.	Data from Ref.	$\bar{u}_C/u_G$	R_C	ρ_C/ρ_G*	$\sigma_{eff} = 1·0$ $\Lambda\|_{x/y_c=100}$	$\sigma_{eff} = 1·0$ D_{max}		$\sigma_{eff} = 0·5$ $\Lambda\|_{x/y_c=100}$	$\sigma_{eff} = 0·5$ D_{max}		$\sigma_{eff} = \Pi_{1·75}^{0·5}$ $\Lambda\|_{x/y_c=100}$	$\sigma_{eff} = \Pi_{1·75}^{0·5}$ D_{max}	
a	7	0·55	1970	1·0	0·030	+0·03	good	0·090	−0·07	fair	0·103	+0·12	fair
b	7	0·76	2620	1·0	0·034	±0·03	good	0·143	−0·15	poor	0·037	+0·04	good
c	7	1·23	4170	1·0	0·024	+0·02	good	0·090	−0·10	fair	0·075	+0·10	fair
d	7	1·85	6330	1·0	0·063	+0·07	fair	0·055	−0·05	fair	0·122	+0·08	poor
e	25	0·76	5570	1·0	0·099	+0·11	fair	0·048	−0·03	good	0·174	+0·22	poor
f	25	2·30	17400	1·0	0·107	+0·14	fair	0·024	±0·02	good	0·176	+0·24	poor
g	2	0·67	5670	≈1·0	0·038	−0·06	good	0·153	−0·17	poor	0·043	+0·07	good
h	11	0·36	4300	≈1·0	0·091	+0·10	fair	0·025	±0·03	good	0·148	+0·17	poor
i	3	0·88	4580	≈1·0	0·021	±0·03	good	0·102	−0·12	fair	0·080	+0·08	fair
j	7	0·58	5150	4·17	0·012	±0·02	good	0·112	−0·15	poor	0·012	+0·05	good
k	7	1·65	14250	4·17	0·018	−0·07	good	0·093	−0·15	fair	0·048	±0·02	good
l	10	1·01	9800	4·17	0·013	−0·05	good	0·085	−0·13	fair	0·027	+0·03	good
m	10	1·01	1990	1·38	0·028	±0·03	good	0·080	−0·15	fair	0·087	+0·08	fair
				$\bar{\Lambda}$	0·054			0·112			0·101		

Ratings: $\Lambda < 0·05$ good; $0·05 < \Lambda < 0·11$ fair; $0·11 < \Lambda$ poor.

The results briefly reported in Table 1 have not been shown in diagrammatic form for reasons of space. Similarly, comparisons between velocity and temperature/profile measurements, integral properties and wall-shear stress are not shown here. They are exhibited in [8] and confirm the conclusion drawn above.

The results discussed in the previous paragraphs are relevant to zero-pressure-gradient situations. The measurements described in [13] permit the prediction procedure to be tested in constant-density, pressure gradient situations. The results showed that the quality of effectiveness predictions was excellent for an adverse pressure gradient ($K_p \simeq -1 \times 10^{-6}$) but poor for a favourable pressure gradient ($K_p \simeq 1{\cdot}8 \times 10^{-6}$). The reason for poor predictions in a strong favourable pressure gradient is the form of the law of the wall used in the prediction procedure. This law of the wall is based on van Driest's hypothesis [28] and is invalid when the flow tends to laminarise as it will for values of K_p larger than approximately 2×10^{-6}. The measurements of velocity profiles and shear stress obtained in the favourable pressure gradient ($K_p \simeq 1{\cdot}8 \times 10^{6}$) showed that this was not a fully turbulent boundary layer and, hence, it is to be expected that the wall law would be inappropriate: consequently, the predictions of velocity and concentration are poor.

3.2 *The upstream region*

The results presented in [6] and [9] revealed and quantified the influence of slot geometry on the impervious-wall effectiveness for constant density flow. Subsequently, [10] extended this work to include the effect of density gradients. The measurements described in [21] and [22] demonstrated the elliptic nature of the flow in the upstream region and also showed that a thick slot lip gave rise to a preferred frequency which was related to the Strouhal frequency. Thus the flow in the upstream region is elliptic and may, in a limited region, be unsteady. Consequently, the use of a prediction method based on parabolic equations is an approximation and can be justified only if it is economical of computer time, easy to use and correctly predicts the important variables. The present procedure is economical (approximately 15 s of IBM 360 time to predict effectiveness from x/y_C of 0–200), easy to use and it remains to modify the specification of effective viscosity to permit correct predictions.

Figure 1 shows with a dotted line the distribution of effective diffusivity corresponding to the mixing-length specification of equation (7) and

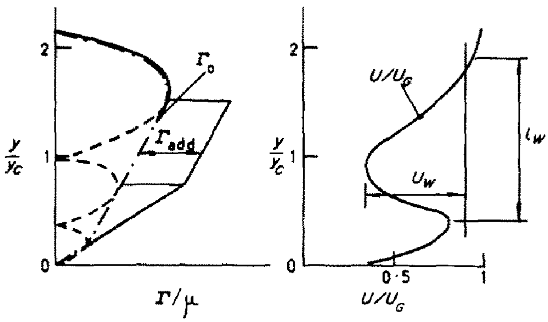


FIG. 1. Effective diffusivity profile used for prediction of the influence of lip thickness on effectiveness.

a realistic mean-velocity profile corresponding to a short distance from the slot exit. The chain dotted line shows the distribution of effective diffusivity used in the calculations of the previous subsection, i.e. zero effective diffusivity is discounted by bridging. The full line shows the form of effective diffusivity distribution used to allow the procedure to predict the influence of lip thickness. Thus Γ_0 represents the effective viscosity or diffusivity profile resulting from the Prandtl mixing-length hypothesis and modified by the bridging procedure. The additive diffusivity, Γ_{add}, is imposed to represent the effect of lip thickness and was computed from a form of eddy-viscosity hypothesis suggested by Prandtl [29] for free flows, i.e.

$$\Gamma_{add} = \frac{\xi}{\sigma_{eff}} \rho l_W u_W \qquad (10)$$

where ξ is specified empirically, l_W is a characteristic width and u_W a characteristic velocity of the wake. In the region between the two inner peaks of the diffusivity profile, the diffusivity was assumed to vary linearly from the value at the inner peak to the augmented value at the adjacent peak. Thus, the resulting eddy-diffusivity profile is continuous across the layer and exhibits an increased value in the wake region behind a slot lip. The additive term, Γ_{add}, decreases to zero as the wake disappears.

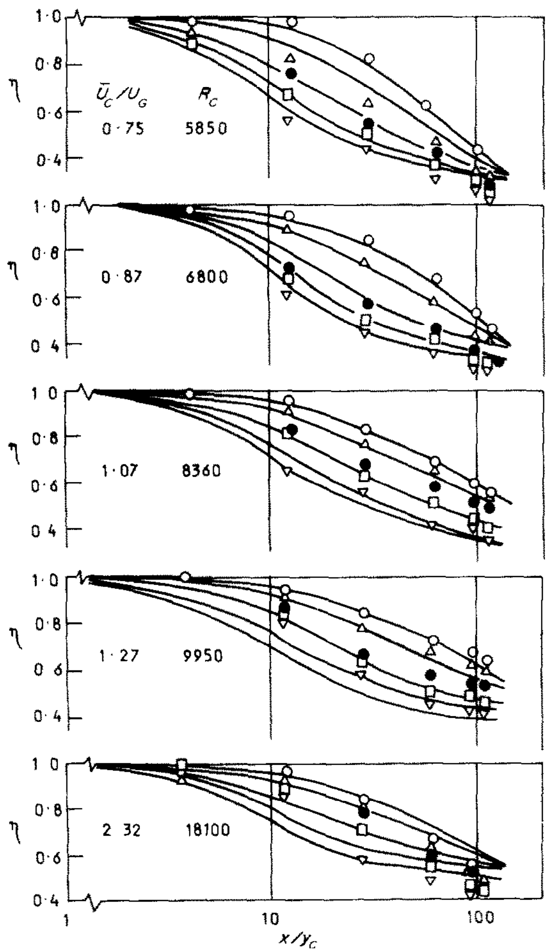


FIG. 2. Predicted and measured impervious-wall effectiveness: influence of slot lip thickness. Data of [9], $\rho_C/\rho_G = 1$.

	t/y_C
○	0·13
△	0·38
●	0·63
□	0·89
△	1·14

The particular form of Γ_{add} used in the present paper may be written as:

$$\Gamma_{add} = 0{\cdot}28\,(t/y_C)^2 \frac{\rho}{\sigma_{eff}} l_W u_W \qquad (11)$$

with ρ taken as local density, l_W as the distance in between two points near the edges of the wake region where the velocity differs from u_G and u_{max} by 1 per cent and u_W is the velocity difference between the velocity minimum in the wake and the mean of u_G and u_{max}. The specification of ξ implied by equation (11) is based on the measured influence of t/y_C on effectiveness [9]: the validity of this empirical procedure may be judged from Fig. 2.

The specification of eddy diffusivity from the mixing-length assumption of equation (8) together with equation (11) is, therefore, capable of predicting the influence of velocity ratio and lip thickness on the adiabatic-wall effectiveness. The ranges of $\bar{u}_C/u_G$ and t/y_C for which this specification is valid is particularly relevant to those found in gas-turbine combustion-chamber practice. It has, however, so far only been tested in constant-density situations whereas combustion chamber practice involves density ratios of the order 2·5–3. Nevertheless, the quality of predictions obtained, with a simple eddy-diffusivity specification, in constant-density flows is encouraging and justifies the predictions of the next section which include variable-density situations.

4. PREDICTIONS FROM THE SLOT EXIT

The predictions shown in Figs. 3 and 4 were obtained with a value of Γ_{add} of zero and show, respectively, the impervious-wall effectiveness for uniform and non-uniform density flows. The data, in both cases, were taken from [7] and were obtained with a tapered slot lip which, for present purposes, can be considered as a lip of zero thickness. The agreement between prediction and measurement is, on the whole.

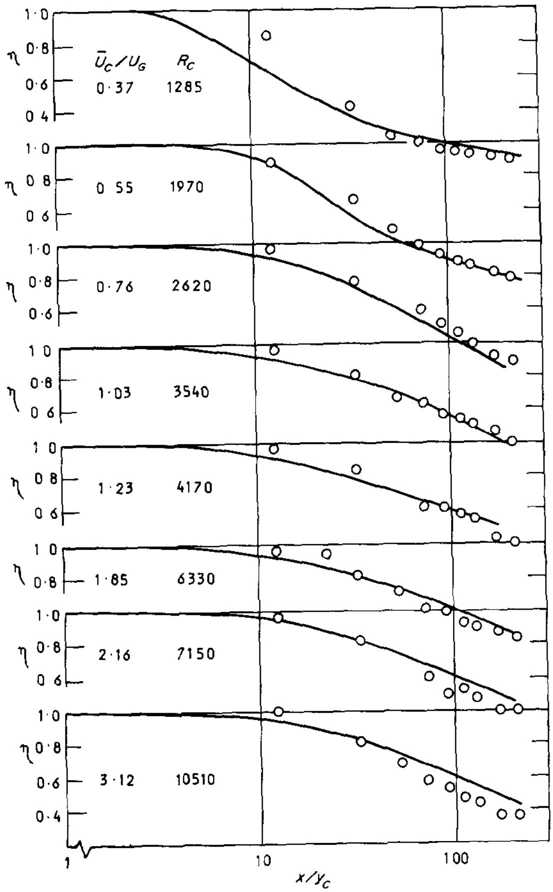
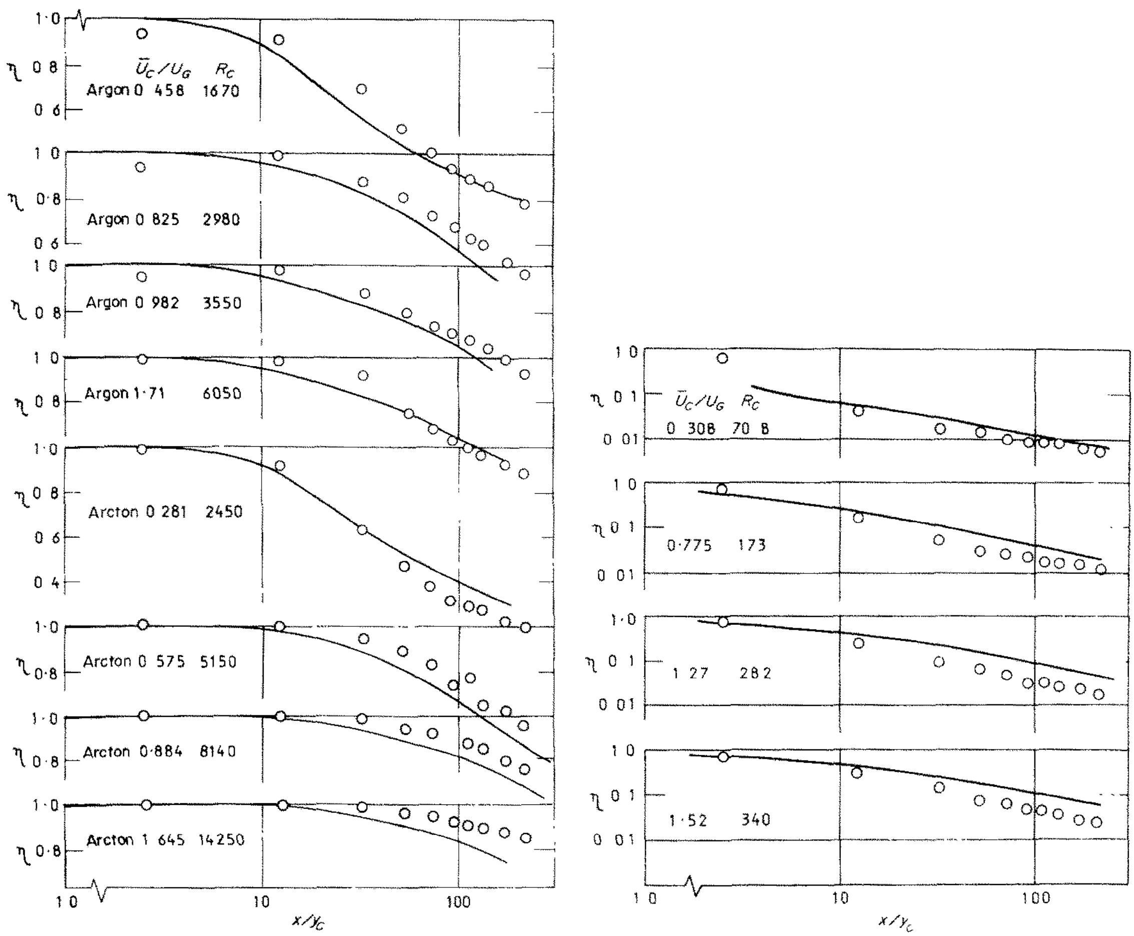


FIG. 3. Predicted and measured impervious-wall effectiveness. Data of [7], $\rho_C/\rho_G = 1$, t/y_C = small.



(a) $\rho_C/\rho_G > 1$. Argon and Arcton 12 injection.

(b) $\rho_C/\rho_G < 1$. Hydrogen injection.

FIG. 4. Predicted and measured impervious-wall effectiveness.
Data of [7], t/y_C = small.

good although some discrepancies can be observed. For the non-uniform density situation ($\rho_C/\rho_G = 4{\cdot}17$) closest to gas turbine practice, the predictions tend to be pessimistic at the higher velocity ratios.

Data from other authors are compared with predictions on Fig. 5. These predictions cover

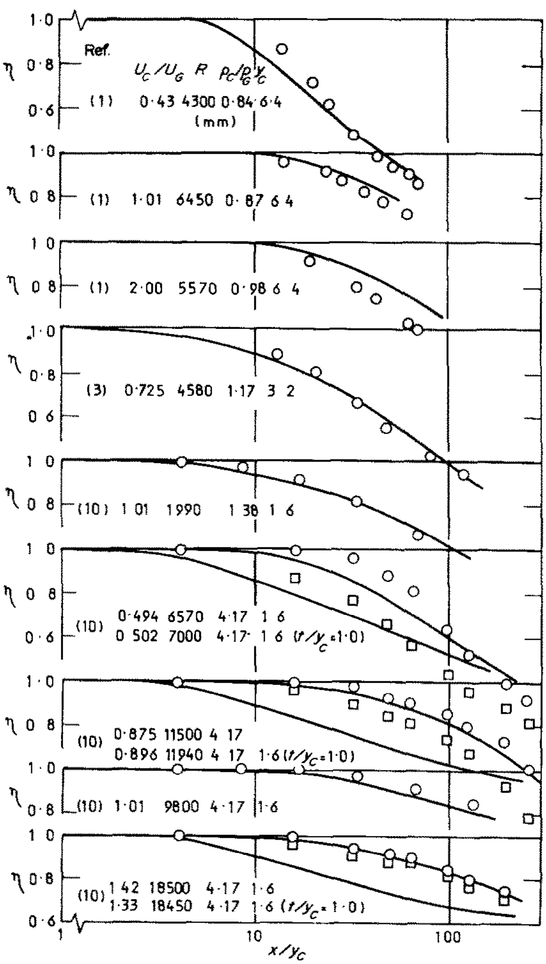


FIG. 5. Predicted and measured effectiveness. Data of [1, 3, 10] $0{\cdot}84 < \rho_C/\rho_G < 4{\cdot}17$, t/y_C = small in all but two cases.

a wide range of velocity and density ratios and include the combined influence of density ratio and lip thickness. Once again, the predictions are generally good but at a density ratio of 4·17, the influence of t/y_C is over-estimated. The predicted influence of t/y_C is shown more clearly on Fig. 6 for two velocity ratios and at one value of x/y_C. The trends are certainly borne out by experiment [10] but the absolute magnitude is still open to question.

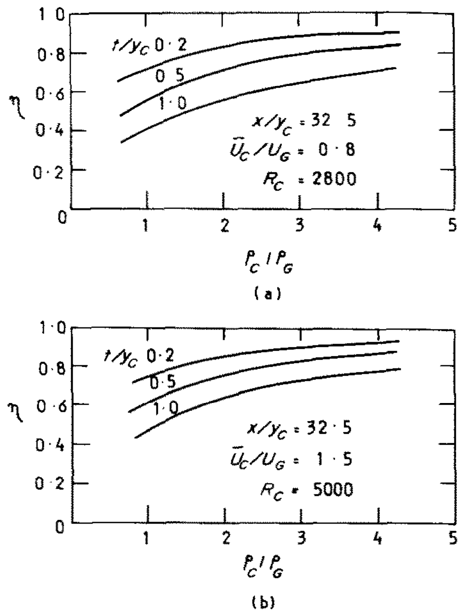


FIG. 6. Predicted influence of density and lip thickness on effectiveness.

Figure 7 returns to constant density situations but shows predictions of Nusselt number in addition to effectiveness. The measurements were obtained in an axisymmetric rig which is described in detail in [8]. The non-adiabatic surface was constructed from 126μ stainless-steel sheet bonded to a Tufnol pipe of 73 mm i.d. and 150 mm long. Low voltage, high current a.c. electricity was passed through the stainless-steel sheet and the power and surface temperature distribution recorded. The heat balance equation has the form:

$$\dot{q}''_{\text{gen}} = h_1(T_W - T_{\text{ad},W}) + h_2(T_W - T_G)$$

with h_1 and $T_{\text{ad},W}$ as unknowns. For each velocity ratio, experiments were carried out for two values

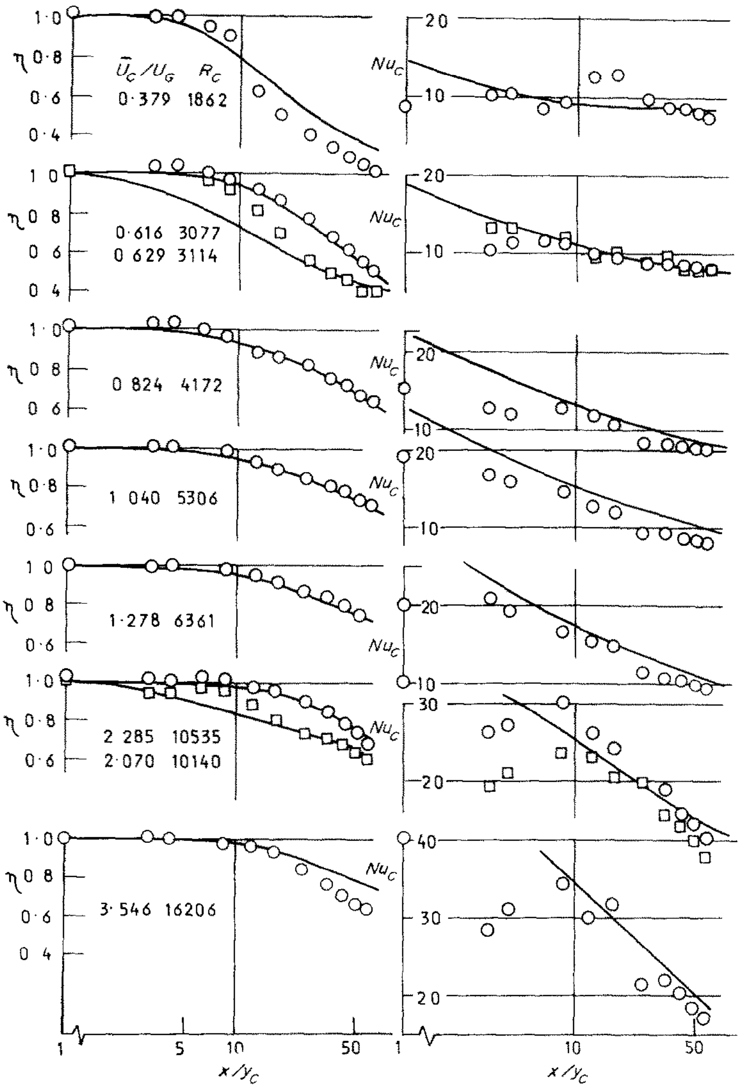


FIG. 7. Predicted and measured adiabatic-wall effectiveness and Nusselt number. Data of [8]. $\rho_C/\rho_G = 0{\cdot}92$.

	t/y_C
○	0·35
□	1·0

of $\dot{q}''_{gen}$ (equal to zero and one non-zero value), the two unknowns were obtained and both the Nusselt number and adiabatic-wall effectiveness determined. The predictions of effectiveness are again satisfactory but the influence of t/y_C is slightly overestimated. The heat transfer predictions are also well within the experimental scatter and, in accord with the experimental results, demonstrate that the influence of t/y_C is not significant for the values of x/y_C greater than 10: for smaller values of x/y_C the measured values of Nusselt number are influenced by the value of t/y_C but the predictions are not.

Before considering the application of the present procedure to the design of combustion-chamber, film-cooling slots, it is useful to review the capabilities and limitations of the procedure, as indicated in the previous paragraphs; this may be accomplished with the aid of Fig. 8. This figure shows that the influences on the adiabatic-wall effectiveness of velocity ratio, density ratio, pressure gradient, slot-lip thickness and lip-boundary-layer thickness are, with few limitations, well predicted by the present procedure. Limitations include the influence of very strong favourable pressure gradients, where the boundary layer tends to laminarise; the influence of density ratios for low values, where the secondary flow is laminar; the combined influence of large values of density ratio, lip thickness and velocity ratio where the present procedure appears to under-predict. The procedure also satisfactorily predicts the influence of velocity ratio and slot-lip thickness on the Nusselt number, except in the region very close to the slot exit.

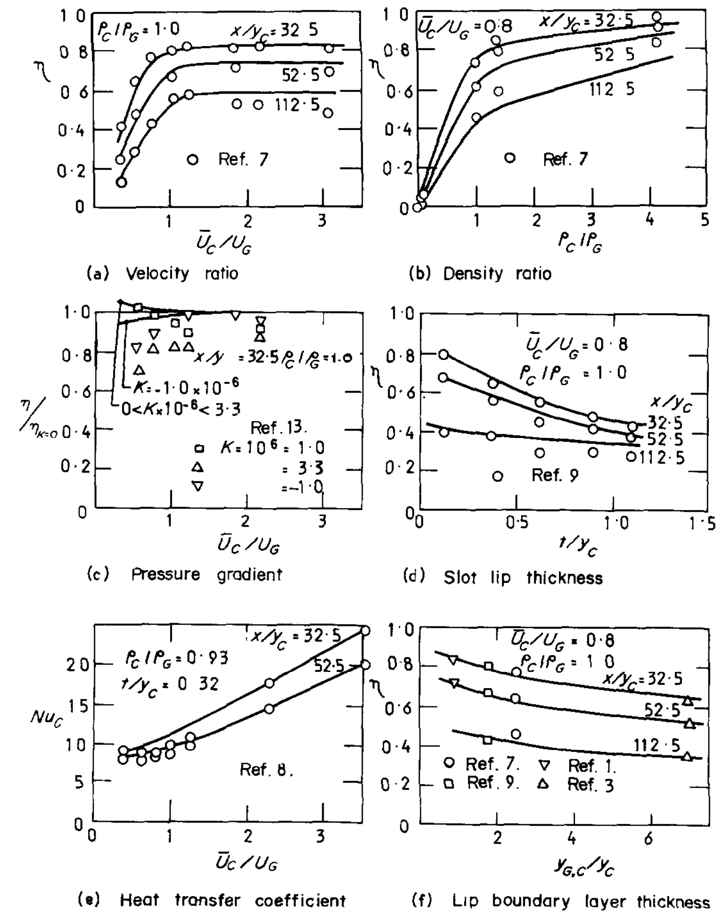


FIG. 8. Summary of comparisons of predicted and measured effectiveness and Nusselt number.

5. APPLICATION TO COMBUSTER WALL COOLING

So far the present paper has been concerned with film cooling through unobstructed two-dimensional slots in low turbulence wind tunnels in the absence of combustion. Under these conditions, it is hoped that the present procedure can be seen to be more general and more accurate than any existing procedure. It is important, however, to examine the conditions under which film-cooling slots operate in practice, for example in gas-turbine combustion chambers. The object of the present discussion is two-fold. First, to place the thermal aspects of film-cooling in perspective by identifying the importance of the various parameters involved, and second, to indicate whether or not the procedure presented in the previous sections can be used under practical circumstances and, if not, what modifications are required.

It must be clearly understood that the flow inside the flame tube of a gas-turbine combustion chamber is three-dimensional, due to asymmetry and irregularities in the geometry and pressure field, and that periodicity exists due to instabilities in the recirculating flow pattern. These properties of the flow would seem to prohibit the use of the two-dimensional prediction procedure. However, a short distance downstream of the film-cooling slot, the flow is likely to be locally two-dimensional and the instabilities are unlikely to seriously influence time-averaged wall temperature. Of course, it is necessary to provide a value of the local free-stream velocity and temperature and of the longitudinal pressure gradient; in practice, values of free-stream velocity are found from measurement and, in the radial direction, are constant only over a short region.

The temperature assumed by the flame tube is such that the heat received by it through radiation and convection from the interior of the chamber is balanced by the heat loss to the surroundings by convection and radiation. For practical purposes, the following equation represents the heat balance:

$$\underbrace{\sigma_B(1+\varepsilon_W/2)\varepsilon_G T_G^{1\cdot 5}\,(T_G^{2\cdot 5}-T_W^{2\cdot 5})}_{R_1} + \underbrace{h_1(T_{\mathrm{ad},W}-T_W)}_{C_1}$$
$$= \underbrace{\sigma_B[1/(2/\varepsilon_W-1)](T_W^4-T_C^4)}_{R_2} + \underbrace{h_2(T_W-T_C)}_{C_2} \qquad (12)$$

The major simplification in the above equation is contained in the gas radiation term, R_1. The derivation of this term is discussed in [30] and assumes, among other things, that: that:

$$\frac{\alpha_W}{\varepsilon_G} = \left(\frac{T_G}{T_W}\right)^{1\cdot 5}$$

where ε_G is the flame emissivity at flame temperature and α_W is the flame absorptivity at wall temperature. The effects of reflection and re-radiation at the wall are approximately allowed for and, for simplicity, the equation assumes the equality of the emissivities of the flame tube and the outer casing and that the outer casing is at a temperature T_C.

Some idea of the relative importance of the seven quantities in equation (12) which influence the wall temperature, i.e. $T_{\mathrm{ad},W}$ (determined by the adiabatic-wall effectiveness, η), T_G and T_C, h_1, h_2, ε_G and ε_W, may be determined from Fig. 9. The abscissa at the bottom of the figure indicates the values of these variables as a fraction of the datum and the corresponding dimensional values are shown by the scales at the top. The datum values chosen may be considered representative of conditions existing at some point within a modern high-compression ratio aero-engine. The wall temperature was computed from equation (12) by an iterative solution procedure. It can be seen from Fig. 9 that, in order to predict the wall temperature to within $\pm 20°C$, the effectiveness, free-stream temperature and coolant temperature must be known, to an accuracy of better than ± 6 per cent. It may readily be appreciated that, for example, the value of T_G appropriate to the radiation term cannot be known to this order of precision. Also, although the predictions contained in the previous section

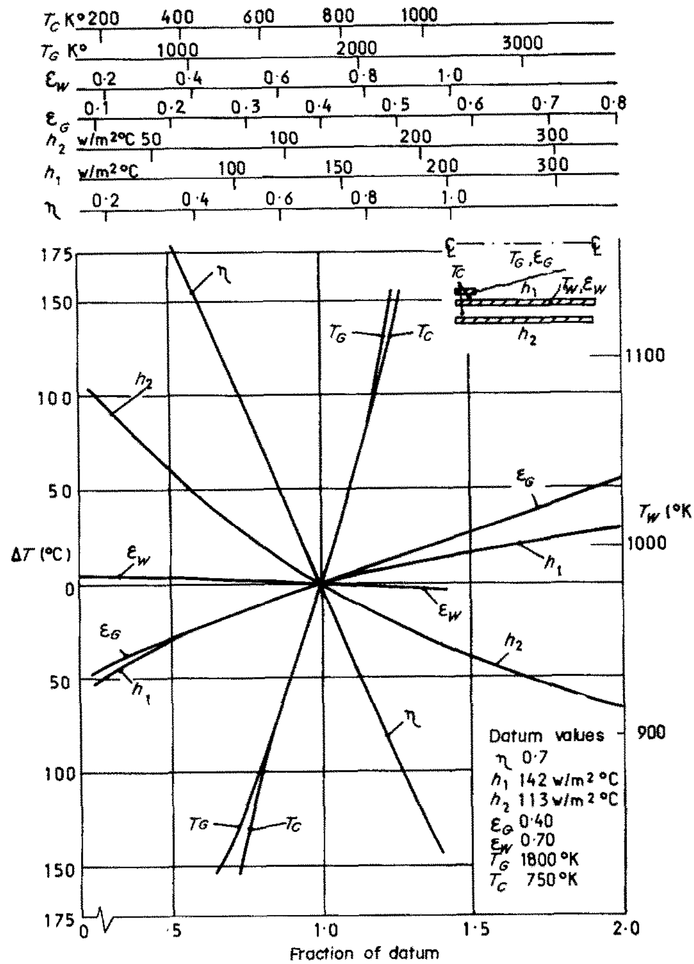


FIG. 9. Influence of η, h_1, h_2, ε_G, ε_N, T_G and T_C on the temperature of a film-cooled combuster surface.

indicate that this is possible for two-dimensional slots, it is unlikely that the value of η will be known to this precision for three-dimensional slots. Consequently, prediction procedures are unlikely to be used to predict the absolute magnitude of the wall temperature but rather the trend which results from the variation of a particular variable.

The prediction of each of these seven factors will be now briefly discussed.

The adiabatic-wall effectiveness and the two convective heat-transfer coefficients can, in principle, be obtained from the prediction procedure described earlier in this paper. This procedure has been shown to provide reasonably good predictions for two-dimensional slots. However, the slots used in practice are not two-dimensional and are subject to a significant and complex geometry effect. The present procedure requires modification before it can predict the influence of three-dimensional geometries. On the other hand, the three-dimensional character of the flow downstream of three-dimensional slots appears to disappear very rapidly and it is likely that a suitable modification to the upstream viscosity law can be found; such a modification depends on the availability of reliable experimental information and this is not presently available. The film heat-transfer coefficient appears to be a weak function of lip thickness and may also prove to be a weak function of three-dimensional geometry; this remains to be proved. The heat-transfer coefficient on the outer surface of the flame tube can be predicted by the present procedure except in the region of the dilution holes.

The next important factor is the flame emissivity. There is considerable uncertainty in its value, especially at high pressure, and the only evaluation method known to the authors is that attributed to Reeves in [30]. It is conceivable that the value given by the empirical expression may be in error by as much as 100 per cent; in this case, the resulting error in wall temperature would be 55°C for the datum conditions of Fig. 9.

Finally, the two gas temperatures must be known accurately. The compressor delivery temperature is probably a good approximation to the coolant temperature for the cooling strips near the primary zone and further approximations can readily be made downstream. Estimation of the flame temperature, on the other hand, is fraught with uncertainty. The practice of obtaining the flame temperature from the overall air–fuel ratio and an assumption of a combustion efficiency is likely to be too crude. In any case, the problem still remains as to which spatial gas temperature is appropriate.

6. CLOSING REMARKS

The problem of predicting the temperature of a film-cooled surface has two main aspects. First, the accurate determination of the conditions in the vicinity of the film-cooled surface, such as T_G, ε_G, T_C, u_C and u_G. Second, the development of an analytical method to predict quantities such as η, h_1 and h_2. Towards the latter aim, sections 3 and 4 of this paper have demonstrated that the present procedure provides a basis for predicting the performance of unobstructed, two-dimensional film-cooling slots over a practically useful range of variables such as $\bar{u}_C/u_G$, ρ_C/ρ_G, dp/dx, t/y_C and $y_{G,C}/y_C$.

This procedure must now be extended to permit satisfactory predictions downstream of practical slot geometries. This may be accomplished, for example, by comparing the predictions of the present procedure with experimental data for practical slot geometries to determine the two-dimensional equivalents (i.e. values of t/y_C and y_C) of the three-dimensional slots. This extension requires the availability of reliable data for practical slot geometries; since insufficient data of this type are presently available the proposed extension of the prediction procedure must be preceded by a programme to provide the required measurements.

REFERENCES

1. R. A. SEBAN, Heat transfer and effectiveness for a turbulent boundary layer with tangential fluid injection, *J. Heat Transfer* **32**, 303 (1960).
2. J. P. HARTNETT, R. C. BIRKEBACK and E. R. G. ECKERT, Velocity distributions, temperature distributions, effectiveness and heat transfer for air injection through a tangential slot into a turbulent boundary layer, *J. Heat Transfer* **83**, 293 (1961).
3. A. E. SAMUEL and P. N. JOUBERT, Film cooling of an adiabatic flat plate in zero pressure gradient in the presence of a hot mainstream and cold tangential secondary injection, A.S.M.E. Paper no. 64 WA/HT-48.
4. R. J. GOLDSTEIN, R. E. RASK and E. R. G. ECKERT, Film cooling with helium injection into an incompressible air flow, *Int. J. Heat Mass Transfer* **9**, 1341 (1966).
5. L. W. CARLSON and E. TALMOR, Gaseous film cooling at various degrees of hot gas acceleration and turbulence levels, *Int. J. Heat Mass Transfer* **11**, 1695 (1968).
6. S. C. KACKER and J. H. WHITELAW, The effect of slot height and slot turbulence intensity on the effectiveness of the uniform-density, two-dimensional wall jet, *J. Heat Transfer* **90**, 469 (1968).
7. B. R. PAI and J. H. WHITELAW, The influence of density gradients on the effectiveness of film cooling, A.R.C. C.P. 1013 (1968).
8. B. R. PAI, The effectiveness of film cooling, Ph.D. thesis, London University (1969).
9. S. C. KACKER and J. H. WHITELAW, An experimental investigation of the influence of slot-lip-thickness on the impervious-wall effectiveness of the uniform density,

two-dimensional wall jet, *Int. J. Heat Mass Transfer* **12**, 1196 (1969).
10. W. K. BURNS and J. L. STOLLERY, The influence of foreign gas injection and slot geometry on the film cooling effectiveness, *Int. J. Heat Mass Transfer* **12**, 935 (1969).
11. R. A. SEBAN and L. H. BACK, Effectiveness and heat transfer for a turbulent boundary layer with tangential injection and variable free-stream velocity, *J. Heat Transfer* **84C**, 235 (1962).
12. J. P. HARTNETT, R. C. BIRKEBACK and E. R. G. ECKERT, Velocity distributions, temperature distributions, effectiveness and heat transfer in cooling a surface with a pressure gradient, *Int. Dev. Heat Transfer* Part IV, 682 (1961).
13. B. R. PAI and J. H. WHITELAW, The influence of strong pressure gradients on film cooling, Proceedings of 1970 Int. Heat Transfer Conference, Paris, Paper FC1.11 (1970).
14. M. P. ESCUDIER and J. H. WHITELAW, The influence of strong adverse pressure gradients on the effectiveness of film cooling, *Int. J. Heat Mass Transfer* **11**, 1289 (1968).
15. J. H. CHIN, S. C. SKIRVIN, L. E. HAYES and A. H. SILVER, Adiabatic wall temperatures downstream of a single tangential injection slot, A.S.M.E. Paper no. 58-A-107.
16. R. A. SEBAN, Effects of initial boundary-layer thickness on a tangential injection system, *J. Heat Transfer* **82**, 392 (1960).
17. S. C. KACKER and J. H. WHITELAW, The dependence of the impervious-wall effectiveness of a two-dimensional wall-jet on the thickness of the upper lip boundary layer, *Int. J. Heat Mass Transfer* **10**, 1623 (1967).
18. S. C. KACKER and J. H. WHITELAW, The prediction of wall-jet and wall-wake flows, *J. Mech. Engng Sci.* **12**, 404 (1970).
19. D. B. SPALDING, Prediction of adiabatic wall temperatures in film cooling systems, *AIAA Jl* **3**, 965 (1965).
20. W. B. NICOLL and J. H. WHITELAW, The effectiveness of the uniform density, two-dimensional wall jet, *Int. J. Heat Mass Transfer* **10**, 623 (1966).
21. S. C. KACKER, W. B. NICOLL and J. H. WHITELAW, An improved method for predicting the effectiveness of the uniform-density, two-dimensional wall jet, Mech. Eng. Dept., Imperial College, Report TWF/TN/30 (1967).
22. S. C. KACKER, The influence of slot geometry on the two-dimensional, turbulent wall jet, Ph.D. thesis, University of London (1969).
23. S. C. KACKER and J. H. WHITELAW, Some properties of the two-dimensional turbulent wall-jet in a moving stream, *J. Appl. Mech.* **35E**, 641 (1968).
24. D. B. SPALDING and S. V. PATANKAR, *Heat and Mass Transfer in Boundary Layers*. Morgan-Grampian Press (1967).
25. S. C. KACKER, B. R. PAI and J. H. WHITELAW, The prediction of wall-jet flows with particular reference to film cooling. *Prog. Heat Mass Transfer*, Vol. 2, p. 163. Pergamon Press, Oxford (1969).
26. B. R. PAI and J. H. WHITELAW, Simplification of the razorblade technique and its application to the measurement of wall-shear stress in wall-jet flows, *Aeronaut. Q.* **20**, 355 (1969).
27. D. B. SPALDING, Heat and mass transfer in boundary layers: Part III—A newer version of the method, Mech. Eng. Dept., Imperial College, Report BL/TN/G/15 (1969).
28. E. R. VAN DRIEST, On turbulent flow near a wall, *J. Aeronaut. Sci.* **23**, 1007 (1956).
29. L. PRANDTL, Über die ausgebildete Turbulenz, *ZAMM* **5**, 136 (1925).
30. A. H. LEFEBVRE and M. V. HERBERT, Heat transfer process in gas turbine combustion chambers. *Proc. Inst. Mech. Engrs* **174**, 12 (1960).

LA DÉTERMINATION DE LA TEMPÉRATURE PARIÉTALE EN PRÉSENCE D'UN FILM REFROIDISSANT

Résumé—Une procédure a été développée pour évaluer la température de paroi adiabatique et le coefficient de convection thermique en aval de fentes bidimensionnelles alimentant un film refroidissant. Des comparaisons entre valeurs calculées et valeurs mesurées démontrent que la méthode prédit convenablement l'influence de la vitesse, du gradient de densité, du gradient longitudinal de pression, du rapport de l'épaisseur des lèvres à la hauteur de la fente. La procédure utilise une forme modifiée du schéma de Spalding–Patankar pour résoudre les équations de couche limite bidimensionnelle et associe des formes particulières de la longueur de mélange selon Prandtl et du nombre de Prandtl effectif.

Bien que la procédure soit satisfaisante pour des configurations de fente bidimensionnelle avec des conditions d'écoulement contrôlé, il est montré que la prédiction des distributions de température de paroi de chambre de combustion n'est pas encore atteinte.

DIE BESTIMMUNG DER WANDTEMPERATUR BEI FILM-KÜHLUNG

Zusammenfassung—Es wurde ein Verfahren zur Bestimmung der adiabaten Wandtemperatur und des Wärmeübergangskoeffizienten in Strömungsrichtung bei zweidimensionalen filmgekühlten Rillen entwickelt. Vergleiche mit berechneten und gemessenen Werten zeigen, dass die Methode den Einfluss von Geschwindigkeitsverhältnis, Dichtegradient, Druckgradient in Längsrichtung, Grenzschichtdicke am

Rillenrand und des Verhältnisses der Randbreite zur Rillentiefe zufriedenstellend berücksichtigt. Das Verfahren benützt eine abgeänderte Form des Spalding–Pantankar-Schemas zur Lösung von zweidimensionalen Grenzschichtgleichungen und umfasst Sonderformen der Prandtlschen Mischlänge und der effektiven Prandtl-Zahl. Obwohl das Verfahren für zweidimensionale Rillenanordnungen mit kontrollierten Fliessbedingungen genügt, ist eine Vorausbestimmung der Wandtemperaturverteilung bei Verbrennung noch nicht möglich.

РАСЧЕТ ТЕМПЕРАТУРЫ СТЕНКИ ПРИ ПЛЕНОЧНОМ ОХЛАЖДЕНИИ

Аннотация—Предлагается метод расчета адиабатической температуры стенки и коэффициента теплообмена вниз по потоку плоских щелей при пленочном охлаждении. Сравнение рассчитанных и измеренных величин показывает, что метод удовлетворительно учитывает влияние относительной скорости, градиента плотности, продольного градиента давления, толщины пограничного слоя у кромки щели и отношения толщины кромки к высоте щели. В расчете используется преобразованная схема Сполдинга-Патанкара для решения уравнений плоского пограничного слоя, в которой учитываются *Pr* эфф и значения длины перемешивания Прандтля. Несмотря на то, что предлагаемый метод дает приемлемые результаты для расчета плоских щелей при регулируемых условиях течения, он не достаточно отработан для определения профиля температуры на стенке камеры сгорания.